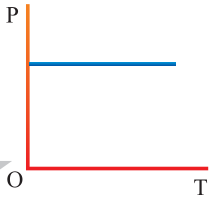


Unit - 2

Kinematic

2.1 Motion in One Dimension : Position

Position of any point is completely expressed by two factors : Its distance from the observer and its direction with respect to observer.

That is why position is characterised by a vector known as position vector.

Let point P is in a *xy* plane and its coordinates are (*x*, *y*). Then position vector $(\vec{r})$ of point will be $x\hat{i} + y\hat{j}$ and if the point P is in a space and its coordinates are (*x*, *y*, *z*) then position vector can be expressed as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

2.2 Rest and Motion

If a body does not change its position as time passes with respect to frame of reference, it is said to be at rest.

And if a body changes its position as time passes with respect to frame of reference, it is said to be in motion.

Frame of Reference : It is a system to which a seet of coordinates are attached and with reference to which observer describes any event.

Rest and motion are relative terms. It depends upon the frame of references.

2.3 Types of Motion

One dimensional	Two dimensional	Three dimensional
Motion of a body in a straight line is called one dimensional motion.	Motion of body in a plane is called two dimensional motion.	Motion of body in a space is called three dimensional motion.
When only one coordinate of the position of a body changes with time then it is said to be moving one dimensionally.	When two coordinates of the position of a body changes with time then it is said to be moving two dimensionally.	When all three coordinates of the position of a body changes with time then it is said to be moving three dimensionally.

e.g., Motion of car on a straight road. Motion of freely falling body.	*e.g.*, Motion of car on a circular turn. Motion of billiards ball.	*e.g.*, Motion of flying kite. Motion of flying insect.

2.4 Distance and Displacement

(1) **Distance** : It is the actual path length covered by a moving particle in a given interval of time.

(i) Its a scalar quantity.

(ii) Dimension : $[M^0 L^1 T^0]$

(iii) Unit : metre (S. I.)

(2) **Displacement :** Displacement is the change in position vector *i.e.*, A vector joining initial to final position.

(i) Displacement is a vector quantity

(ii) Dimension : $[M^0 L^1 T^0]$

(iii) Unit : metre (S. I.)

(iv) If $\overline{S}_1, \overline{S}_2, \overline{S}_3, ..., \overline{S}_n$ are the displacements of a body then the total (net) displacement is the vector sum of the individuals.

$$\overline{S} = \overline{S}_1 + \overline{S}_2 + \overline{S}_3 + ... + \overline{S}_n$$

(3) **Comparison between distance and displacement :**

(i) Distance $\geq$ Displacement.

(ii) For a moving particle distance can never be negative or zero while displacement can be *i.e.*, Distance > 0 but Displacement > = or < 0.

(iii) For motion between two points displacement is single valued while distance depends on actual path and so can have many values.

(iv) For a moving particle distance can never decrease with time while displacement can. Decrease in displacement with time means body is moving towards the initial position.

(v) In general magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without change in direction.

2.5 Speed and Velocity

(1) Speed : Rate of distance covered with time is called speed.

(i) It is a scalar quantity having symbol v.

(ii) Dimension : $[M^0L^1T^{-1}]$

(iii) Unit : metre/second (S.I.), cm/second (C. G. S.)

(iv) Types of speed :

(a) Uniform speed : When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed.

(b) Non-uniform (variable) speed : In non-uniform speed particle covers unequal distances in equal intervals of time.

(c) Average speed : The average speed of a particle for a given 'Interval of time' is defined as the ratio of distance travelled to the time taken.

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}}\ ;\ V_{av} = \frac{\Delta s}{\Delta t}$$

- **Time average speed :** When particle moves with different uniform speed $\upsilon_1, \upsilon_2, \upsilon_3$ .... etc. in different time intervals t_1, t_2, t_3, ... etc. respectively, its average speed over the total time of journey is given as

$$V_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...}$$

$$= \frac{\upsilon_1 t_1 + \upsilon_2 t_2 + \upsilon_3 t_3 + ...}{t_1 + t_2 + t_3 + ...}$$

Special case : When particle moves with speed v_1 upto half time of its total motion and in rest time it is moving with speed v_2 then $V_{av} = \frac{V_1 + V_2}{2}$.

- **Distance averaged speed :** When a particle describes different distances d_1, d_2, d_3, ... with different time intervals t_1, t_2, t_3, ... with speeds v_1, v_2, v_3, ... respectively then the speed of particle averaged over the total distance can be given as

$$V_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...}$$

$$= \frac{d_1 + d_2 + d_3 + ...}{\frac{d_1}{\upsilon_1} + \frac{d_2}{\upsilon_2} + \frac{d_3}{\upsilon_3} + ...}$$

(d) Instantaneous speed : It is the speed of a particle at particular instant. When we say "speed", it usually means instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (*i.e.* $\Delta t \to 0$). Thus

$$\text{Instantaneous speed } v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}.$$

(2) Velocity : Rate of change of position *i.e.*, rate of displacement with time is called velocity.

(i) It is a vector quantity having symbol v.

(ii) Dimension : $[M^0 L^1 T^{-1}]$

(iii) Unit : metre/second (S. I.), cm/second (C. G. S.)

(iv) Types

(a) Uniform velocity : A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line without reversing its direction.

(b) Non-uniform velocity : A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).

(c) Average velocity : It is defined as the ratio of displacement to time taken by the body

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}; \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

(d) Instantaneous velocity : Instantaneous velocity is defined as rate of change of position vector of particles with time at a certain instant of time.

$$\text{Instantaneous velocity } \vec{v} = \lim_{t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}.$$

(v) Comparison between instantaneous speed and instantaneous velocity

(a) Instantaneous velocity is always tangential to the path followed by the particle.

(b) A particle may have constant instantaneous speed but variable instantaneous velocity.

(c) The magnitude of instantaneous velocity is equal to the instantaneous speed.

(d) If a particle is moving with constant velocity then its average velocity and instantaneous velocity are always equal.

(e) If displacement is given as a function of time, then time derivative of displacement will give velocity.

(vi) Comparison between average speed and average velocity

(a) Average speed is scalar while average velocity is a vector both having same units (m/s) and dimensions $[LT^{-1}]$.

(b) Average speed or velocity depends on time interval over which it is defined.

(c) For a given time interval average velocity is single valued while average speed can have many values depending on path followed.

(d) If after motion body comes back to its initial position then $\vec{v}_{av} = \vec{0}$ (as $\Delta\vec{r} = 0$) but $v_{av} > \vec{0}$ and finite as ($\Delta s > 0$).

(e) For a moving body average speed can never be negative or zero (unless $t \to \infty$) while average velocity can be *i.e.*, $\vec{v}_{av} > 0$ while $\vec{v}_{av} =$ or < 0.

2.6 Acceleration

The time rate of change of velocity of an object is called acceleration of the object.

(1) It is a vector quantity. It's direction is same as that of change in velocity (not of the velocity)

(2) There are three possible ways by which change in velocity may occur

When only direction of velocity changes	When only magnitude of velocity changes	When both magnitude and direction of velocity changes
Acceleration perpendicular to velocity	Acceleration parallel or anti-parallel to velocity	Acceleration has two components one is perpendicular to velocity and another parallel or antiparallel to velocity
e.g. Uniform circular motion	*e.g.* Motion under gravity	*e.g.* Projectile motion

(3) Dimension : $[M^0 L^1 T^{-2}]$

(4) Unit : metre/second^2 (S. I.); cm/second^2 (C. G. S.)

(5) Types of acceleration :

(i) Uniform acceleration : A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

- If a particle is moving with uniform acceleration, this does not necessarily imply that particles is moving in straight line, *e.g.*, Projectile motion.

(ii) Non-uniform acceleration : A body is said to have non-uniform acceleration, if magnitude or direction or both, change during motion.

(iii) Average acceleration : $\vec{a}_{av} = \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t}$.

The direction of average acceleration vector is the direction of the change in velocity vector as $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t}$.

(iv) Instantaneous acceleration = $\vec{a} = \lim_{\Delta t \to 0} \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{d\vec{v}}{dt}$.

(v) For a moving body there is no relation between the direction of instantaneous velocity and direction of acceleration.

e.g.(a) In uniform circular motion $\theta = 90°$ always (b) in a projectile motion θ is variable for every point of trajectory.

(vi) By definition, $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{x}}{dt^2} \left[\text{As } \vec{v} = \dfrac{d\vec{x}}{dt}\right]$

(vii) If velocity is given as a function of position, then by chain rule

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v \times \frac{dv}{dx}\left[\text{as } v = \frac{dx}{dt}\right]$$

(viii) If a particle is accelerated for a time t_1 by acceleration a_1 and for time t_2 by acceleration a_2 then average acceleration is

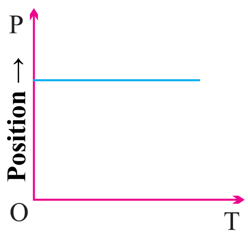
$$a_{av} = \frac{a_1t_1 + a_2t_2}{t_1 + t_2}$$

(ix) Acceleration can be positive, zero or negative. Positive acceleration means velocity increasing with time, zero acceleration means velocity is uniform constant while negative acceleration (retardation) means velocity is decreasing with time.

(x) For motion of a body under gravity, acceleration will be equal to 'g',where g is the acceleration due to gravity. Its normal value is 9.8 m/s^2 or 980 cm/s^2 or 32 feet/s^2.

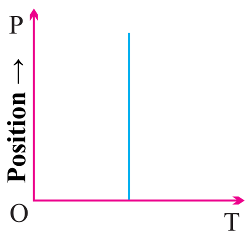
2.7 Position Time Graph

Various position-time graphs and their interpretation



$\theta = 0°$ so $v = 0$

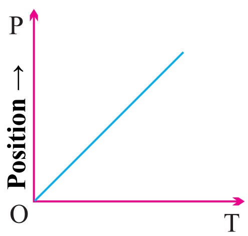
i.e., line parallel to time axis represents that the particle is at rest.



$\theta = 90°$ so $v = \infty$

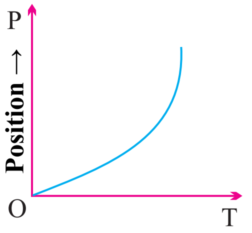
i.e., line perpendicular to time axis represents that particle is changing its position but time does not changes it means the particle possesses infinite velocity.

Practically this is not possible.



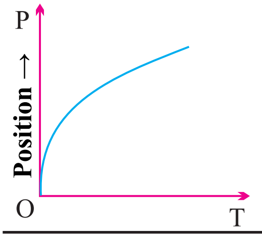
θ = constant so v = constant, $a = 0$

i.e., line with constant slope represents uniform velocity of the particle.



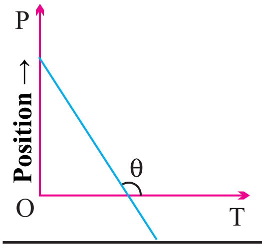
θ is increasing so v is increasing, a is positive.

i.e., line bending towards position axis represents increasing velocity of particle. It means the particle possesses acceleration.



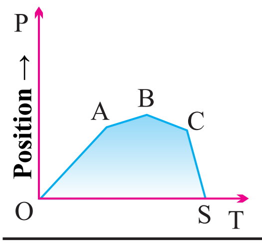
θ is decreasing so v is decreasing, a is negative.

i.e., line bending towards time axis represents decreasing velocity of the particle. It means the particle possesses retardation.

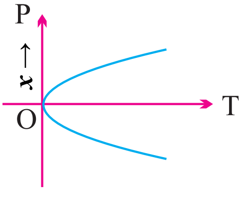


θ constant but > 90° so v will be constant but negative.

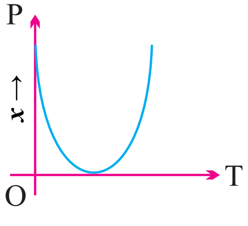
i.e., line with negative slope represent that particle returns towards the point of reference, (negative displacement).



Straight line segments of different slopes represent that velocity of the body changes after certain interval of time.



This graph shows that at one instant the particle has two positions. Which is not possible.



The graph shows that particle coming towards origin initially and after that it is moving away from origin.

Note :

- If the graph is plotted between distance and time then it is always an increasing curve and it never comes back towards origin because distance.
- For two particles having displacement time graph with slope θ_1 and θ_2 possesses velocities v_1 and v_2 respectively then $\dfrac{v_1}{v_2} = \dfrac{\tan\theta_1}{\tan\theta_2}$.

2.8 Velocity Time Graph

The graph is plotted by taking time t along x axis and velocity of the particle on y axis.

Distance and displacement : The area covered between the velocity time graph and time axis gives the displacement and distance travelled by the body for a given time interval.

Then Total distance = Addition of modulus of different area.

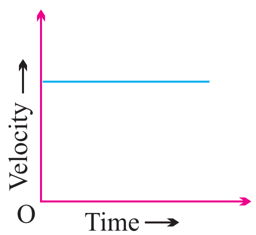
i.e. $\quad s = \int |v| dt$

Total displacement = Addition of different area considering their sign.

i.e., $\quad r = \int v dt$

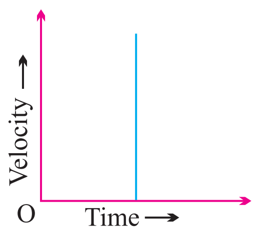
Acceleration : It is clear that slope of velocity-time graph represents the acceleration of the particle.

Various position-time graphs and their interpretation



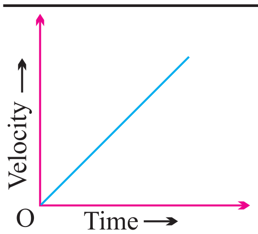
$\theta = 0$, $a = 0$, v = constant

i.e., line parallel to time axis represents that the particle is moving with constant velocity.



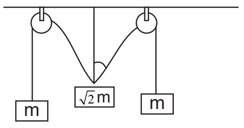
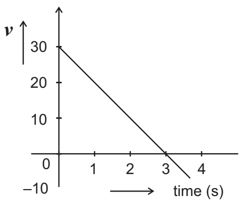
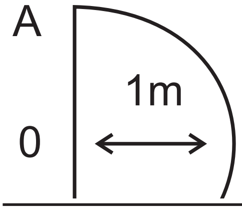
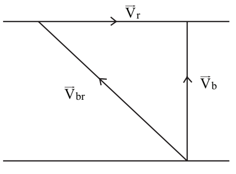
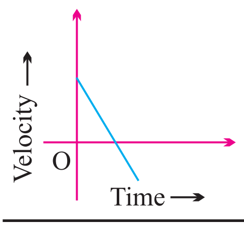
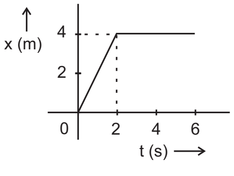
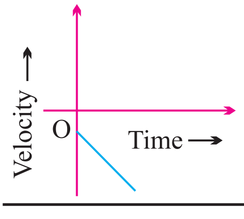
$\theta = 90°$, $a = \infty$, v = increasing

i.e., line perpendicular to time axis represents that particle is increasing its velocity but time does not change. It means the particle possesses infinite acceleration. Practically it is not possible.



θ = constant, so a = constant and v is increasing uniformly with time

i.e., line with constant slope represents uniform acceleration of the particle.

	θ is increasing so acceleration increasing *i.e.*, line bending towards velocity axis represents the increasing acceleration in the body.
	θ is decreasing so acceleration decreasing *i.e.*, line bending towards time axis represents the decreasing acceleration in the body.
	Positive constant acceleration because θ is constant and < 90° but initial velocity of the particle is negative.
	Positive constant acceleration because θ is constant and < 90° but initial velocity of the particle is positive.
	Negative constant acceleration because θ is constant and > 90° but initial velocity of the particle is positive
	Negative constant acceleration because θ is constant and > 90° but initial velocity of the particle is zero.
	Negative constant acceleration because θ is constant and > 90° but initial velocity of the particle is negative.

2.9 Equations of Kinematics

These are the various relations between *u, v, a, t* and *s* for the moving particle where the notations are used as :

u = Initial velocity of the particle at time $t = 0$ sec

v = Final velocity at time t sec

a = Acceleration of the particle

s = Distance travelled in time t sec

s_n = Distance travelled by the body in n^{th} sec

(1) When particle moves with constant acceleration

(i) Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.

(ii) There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

(iii) Equations of motion in scalar form — Equation of motion in vector form

Equations of motion in scalar form	Equation of motion in vector form
$v = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$v^2 = v^2 + 2\,as$	$\vec{v}\,.\,\vec{v} - \vec{u}\,.\,\vec{u} = 2\,\vec{a}\,.\,\vec{s}$
$s = \left(\frac{u+v}{2}\right)t$	$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$
$s_n = u + \frac{a}{2}(2n - 1)$	$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n - 1)$

(2) Important points for uniformly accelerated motion

(i) If a body starts from rest and moves with uniform acceleration then distance covered by the body in t sec is proportional to t^2 (*i.e.*, $s \propto t^2$).

So the ratio of distance covered in 1 sec, 2 sec and 3 sec is $1^2 : 2^2 : 3^2$ or $1 : 4 : 9$.

(ii) If a body starts from rest and moves with uniform acceleration then distance covered by the body in nth sec is proportional to $(2n - 1)$ (*i.e.* $s_n \propto (2n - 1)$).

So the ratio of distance covered in I sec, II sec and III sec is I : 3 : 5.

(iii) A body moving with a velocity u is stopped by application of brakes after covering a distance s. If the same body moves with velocity nu and same braking force is applied on it then it will come to rest after covering a distance of n^2 s.

2.10 Motion of Body Under Gravity (Free Fall)

Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

In the absence of air resistance, it is found that all bodies fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude ($h \ll R$) is called free fall.

An ideal one-dimensional motion under gravity in which air resistance and the small changes in acceleration with height are neglected.

PROJECTILE MOTION

2.11 Introduction

If the force acting on a particle is oblique with initial velocity then the motion of particle is called projectile motion.

2.12 Projectile

A body which is in flight through the atmosphere but is not being propelled by any fuel is called projectile.

2.13 Assumptions of Projectile Motion

(1) There is no resistance due to air.

(2) The effect due to curvature of earth is negligible.

(3) The effect due to rotation of earth is negligible.

(4) For all points of the trajectory, the acceleration due to gravity 'g' is constant in magnitude and direction.

2.14 Principles of Physical Independence of Motions

(1) The motion of a projectile is a two-dimensional motion. So, it can be discussed in two parts. Horizontal motion and vertical motion. These two motions take place independent of each other.This is called the principle

of physical independence of motions.

(2) The velocity of the particle can be resolved into two mutually perpendicular components. Horizontal component and vertical component.

(3) The horizontal component remains unchanged throughout the flight. The force of gravity continuously affects the vertical component.

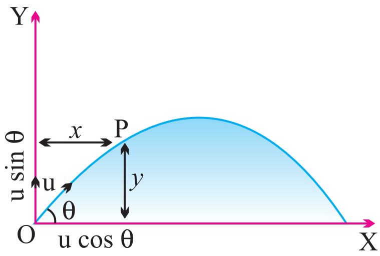
(4) The horizontal motion is a uniform motion and the vertical motion is a uniformly accelerated retarded motion.

2.15 Types of Projectile Motion

(1) Oblique projectile motion (2) Horizontal projectile motion (3) Projectile motion on an inclined plane

2. 16 Oblique Projectile

In projectile motion, horizontal component of velocity (u cos θ), acceleration (g) and mechanical energy remains constant while, speed, velocity, vertical component of velocity (u sin θ), momentum kinetic energy and potential energy all changes. Velocity, and KE are maximum at the point of projection while minimum (but not zero) at highest point.



(1) Equation of trajectory : A projectile thrown with velocity u at an angle θ with the horizontal. The velocity u can be resolved into two rectangular components u cos θ component along X-axis and u sin θ component along Y-axis.

$$y = x \tan\theta - \frac{1}{2}\frac{gx^2}{u^2 \cos^2\theta}$$

Note :

- Equation of oblique projectile also can be written as

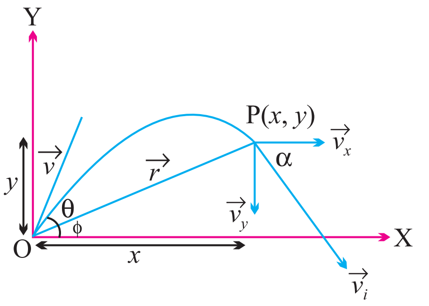
$$y = x \tan\theta\left[1 - \frac{x}{R}\right]$$

(where R = horizonal range)

(2) Displacement of projectile $(\vec{r})$: Let the particle acquires a position

P having the coordinates (x, y) just after time t from the instant of projection.

The corresponding position vector of the particle at time t is $\vec{r}$ shown in the figure.



$$\vec{r} = x\hat{i} + y\hat{j} \quad \text{...(i)}$$

The horizontal distance covered during time t given as

$$x = v_x t \Rightarrow x = u \text{ cor } \theta\, t \quad \text{...(ii)}$$

The vertical velocity of the paticle at time t is given as

$$y = u \sin\theta\, t - \frac{1}{2} gt^2 \quad \text{...(iii)}$$

and $\phi = \tan^{-1}(y/x)$

Note :

- The angle of elevation ϕ of the highest point of the projectile and the angle of projection ϕ are related to each other as $\tan\phi = \frac{1}{2}\tan\theta$.

(3) **Instantaneous velocity *v*** : In projectile motion, vertical component of velocity changes but horizontal component of velocity remains always constant.

Let v_i be the instantaneous velocity of projectile at time t direction of this velocity is along the tangent to the trajectory at point P.

$$\vec{v}_i = v_x\hat{i} + v_y\hat{j} \Rightarrow v_i = \sqrt{v_x^2 + v_y^2}$$

Direction of instantaneous velocity $\tan a\, \alpha = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$.

(7) **Time of flight :** The total time taken by the projectile to go up and come down to the same level from which it was projected is called time of flight.

For vertical upward motion

$$0 = u \sin\theta - gt \Rightarrow t = (u \sin\theta/g)$$

$$\text{Time of flight } T=2t = \frac{2u\sin\theta}{g}.$$

(8) **Horizontal range** : It is the horizontal distance travelled by a body during the time of flight. So by using second equation of motion

$$R = u\cos\theta \times T = u\cos\theta \times (2u\sin\theta/g) = \frac{u^2 \sin 2\theta}{g}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

If angle of projection is changed from θ to $\theta' = (90 - \theta)$ then range remains unchanged. These angles are called complementary angle of projection.

(iv) **Maximum range :** For range to be maximum $\frac{dR}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left[\frac{u^2 \sin 2\theta}{g}\right]$ = 0, a projectile will have maximum range when it is projected at an angle of 45° to the horizontal and the maximum range will be (u^2/g).

When the range is maximum, the height H reached by the projectile

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2 \sin^2 45}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$$

(v) **Relation between horizontal range and maximum height :**

$$R = 4H \cot\theta$$

If $R = 4H \cot\theta = \tan^{-1}(1)$ or $\theta = 45°$.

(9) **Maximum height :** It is the maximum height from the point of projection, a projectile can reach.

So, by using $v^2 = u^2 + 2as$

$$0 = (u\sin\theta)^2 - 2gH$$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

(i) $H_{max} = \frac{u^2}{2g}$ (when $\sin^2\theta = \max = 1$ *i.e.*, $\theta = 90°$)

i.e., for maximum height body should be projected vertically upward.

(10) Motion of a projectile as observed from another projectile is a straight line.

2. 17 Horizontal Projectile

A body be projected horizontally from a certain height 'y' vertically above the ground with initial velocity u. If friction is considered to be absent then there is no other horizontal force which can affect the horizontal motion. The horizontal velocity therefore remains constant.

(4) **Time of flight :** If a body is projected horizontally from a height h with velocity u and time taken by the body to reach the ground is T, then

$$T = \sqrt{\frac{2h}{g}}$$

(5) **Horizontal range :** Let R is the horizontal distance travelled by the body

$$R = u\sqrt{\frac{2h}{g}}$$

(6) If projectiles A and B are projected horizontally with different initial velocity from same height and third particle C is dropped from same point then

(i) All three particles will take equal time to reach the ground.

(ii) Their net velocity would be different but all three particle possess same vertical component of velocity.

(iii) The trajectory of projectiles A and B will be straight line *w.r.t.* particle C.

(7) If various particles thrown with same initial velocity but indifferent direction then

(i) They strike the ground with same speed at different times irrespective of their initial direction of velocities.

(ii) Time would be least for particles which was thrown vertically downward.

(iii) Time would be maximum for particle A which was thrown vertically upward.

CIRCULAR MOTION

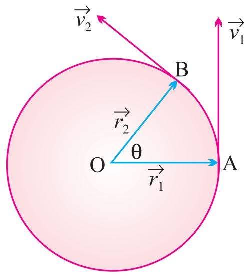
Circular motion is another example of motion in two dimensions. To create circular motion in a body it must be given some initial velocity and a force must then act on the body which is always directed at right angles to instantaneous velocity.

Circular motion can be classified into two types-Uniform circular motion and non-uniform circular motion.

2. 18 Variables of Circular Motion

(1) Displacement and distance : When particle moves in a circular path describing an angle θ during time *t* (as shown in the figure) from the position A to the position B, we see that the magnitude of the position vector $\vec{r}$ (that is equal to the radius of the circle) remains constant, *i.e.*, $|\vec{r_1}| = |\vec{r_2}| = r$ and the direction of the position vector changes from time to time.

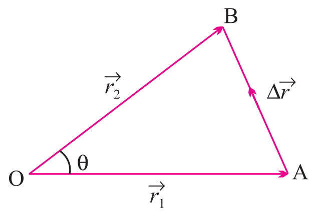
(i) Displacement : The change of position vector or the displacement $\vec{\Delta r}$ of the particle from position A to position B is given by referring the figure.



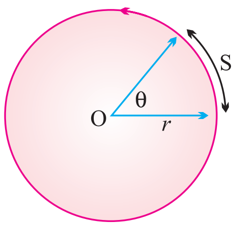
$$\vec{\Delta r} = \vec{r_2} - \vec{r_1}$$

$$\Delta r = 2r \sin \frac{\theta}{2}$$

(ii) Distance : The distance covered by the particle during the time *t* is given as *d* = length of the arc AB



(2) Angular displacement (θ) : The angle turned by a body moving on a circle from some reference line is called angular displacement.



(i) Dimension = $[M^0L^0T^0]$ (as θ = arc/radius).

(ii) Units = Radian or Degree. It is some times also specified in terms of fraction or multiple of revolution.

(iii) 2π rad = 360° = 1 Revolution

(iv) Angular displacement is a axial vector quantity. Its direction depends upon the sense of rotation of the object can be given by Right Hand Rule; which states that if the curvature of the fingers of right hand represents the sense of rotation of the object, then the thumb, held perpendicular to the curvature of the fingers, represents the direction of angular displacement vector.

(v) Relation between linear displacement and angular displacement $\vec{s} = \vec{\theta} \times \vec{r}$ or $s = r\theta$.

(3) Angular velocity (ω) : Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

(i) Angular velocity $\omega = \dfrac{\text{angle traced}}{\text{time taken}} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$

$$\omega = \frac{d\theta}{dt}$$

(ii) Dimension : $[M^0\, L^0\, T^{-1}]$

(iii) Units : Radians per second (rad. s^{-1}) or Degree per second.

(iv) Angular velocity is an axial vector. Its direction is the same as that of $\Delta\theta$.

(v) Relation between angular velocity and linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$.

(vi) For uniform circular motion co remains constant where as for non-uniform motion ω varies with respect to time.

(4) Change in velocity : We want to know the magnitude and direction of the change in velocity of the particle which is performing uniform circular motion as it moves from A to B during time t as shown in figure. The change in velocity vector is given as

$$\Delta\vec{v} = \vec{v_2} - \vec{v_1}$$

$$\Delta v = 2v \sin\frac{\theta}{2}$$

- Relation between linear velocity and angular velocity. In vector form

$$\vec{v} = \vec{\omega} \times \vec{r}.$$

(5) **Time period (T) :** In circular motion, the time period is defined as the time taken by the object to complete one revolution on its circular path.

(6) **Frequency (*n*) :** In circular motion, the frequency is defined as the number of revolutions completed by the object on its circular path in a unit time.

(i) Units : s^{-1} or hertz (Hz).

(ii) Dimension : $[M^0L^0T^{-1}]$

Note :

- Relation between time period and frequency :

  $\therefore \qquad T = 1/n$

- Relation between angular velocity, frequency and time period :

$$\omega = \frac{2\pi}{T} = 2\pi n$$

(7) **Angular acceleration (α) :** Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

(i) $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

(ii) Units : *rad* s^{-2}

(iii) Dimension : $[M^0\ L^0\ T^{-2}]$

(iv) Relation between linear acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$

(v) For uniform circular motion since θ is constant so $\alpha = \frac{d\omega}{dt} = 0$.

(vi) For non-uniform circular motion $\alpha \neq 0$.

2. 19 Centripetal Acceleration

(1) Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.

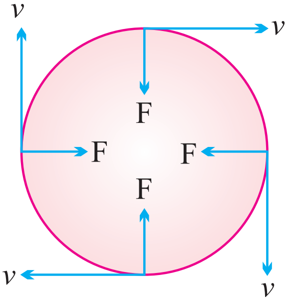
(2) It always acts on the object along the radius towards the centre of the circular path.

(3) Magnitude of centripetal acceleration $\alpha = \frac{v^2}{r} = \omega^2 r = 4\pi n^2\, r = \frac{4\pi^2}{T^2} r$.

(4) Direction of centripetal acceleration : It is always the same as that of $\Delta\vec{v}$.

2. 20 Centripetal Force

According to Newton's first law of motion, whenever a body moves in a straight line with uniform velocity, no force is required to maintain this velocity. But when a body moves along a circular path with uniform speed, its direction changes continuously *i.e.*, velocity keeps on changing on account of a change in direction. According to Newton's second law of motion, a change in the direction of motion of the body can take place only if some external force acts on the body.



Due to inertia, at every point of the circular path; the body tends to move along the tangent to the circular path at that point (in figure). Since every body has directional inertia, a velocity cannot change by itself and as such we have to apply a force. But this force should be such that it changes the direction of velocity and not its magnitude. This is possible only if the force acts perpendicular to the direction of velocity. Because the velocity is along the tangent, this force must be along the radius (because the radius of a circle at any point is perpendicular to the tangent at that point).

Further, as this force is to move the body in a circular path, it must acts towards the centre. The centre-seeking force is called the centripetal force.

Hence, centripetal force is that force which is required to move a body in a circular path with uniform speed. The force acts on the body along the radius and towards centre.

(1) Formulae for centripetal forced :

$$F = \frac{mv^2}{r} = m\omega^2 r = m4\pi^2 n^2 r = \frac{m4\pi^2 r}{T^2}$$

(2) Centripetal force in different situation

Situation	Centripetal Force
A particle tied to a string and whirled in a horizontal circle.	Tension in the string.
Vehicle taking a turn on a level road.	Frictional force exerted by the road on the tyres.
A vehicle on a speed breaker.	Weight of the body or a component of weight.
Revolution of earth around the sun.	Gravitational force exerted by the sun.
Electron revolving around the nucleus in an atom.	Coulomb attraction exerted by the protons in the nucleus.
A charged particle describing a circular path in a magnetic field.	Magnetic force exerted by the agent that sets up the magnetic field.

2. 21 Centrifugal Force

It is an imaginary force due to incorporated effects of inertia. Centrifugal force is a fictitious force which has significance only in a rotating frame of reference.

2.22 Work done by Centripetal Force

The work done by centripetal force is always zero as it is perpendicular to velocity and hence instantaneous displacement.

Example :

(i) When an electron revolve around the nucleus in hydrogen atom in a particular orbit, it neither absorb nor emit any energy means its energy remains constant.

(ii) When a satellite established once in a orbit around the earth and it starts revolving with particular speed, then no fuel is required for its circular motion.

2. 23 Skidding of Vehicle on a Level Road

When a vehicle turns on a circular path it requires centripetal force. If friction provides this centripetal force then vehicle can move in circular path safely if Friction force $\geq$ Required centripetal force

$$\mu mg \geq \frac{mv^2}{r}$$

$\therefore \quad v_{safe} \leq \sqrt{\mu rg}$

This is the maximum speed by which vehicle can turn in a circular path of radius r, where coefficient of friction between the road and tyre is μ.

2. 24 Skidding of Object on a Rotating Platform

On a rotating platform, to avoid the skidding of an object (mass m) placed at a distance r from axis of rotation, the centripetal force should be provided by force of friction.Centripetal force = Force of friction

$$m\omega^2 r = \mu mg$$

$$\therefore \quad \omega_{max} = \sqrt{(\mu g / r)},$$

Hence maximum angular velocity of rotation of the platform is $\sqrt{(\mu g / r)}$, so that object will not skid on it.

2.25 Bending of a Cyclist

A cyclist provides himself the necessary centripetal force by leaning inward on a horizontal track, while going round a curve. Consider a cyclist of weight mg taking a turn of radius r wih velocity v. In order to provide the necessary centripetal force, the cyclist leans through angle θ inwards as shown in figure.

$$R \sin \theta = \frac{mv^2}{r} \quad \text{...(i)}$$

and

$$R \cos \theta = mg \quad \text{....(ii)}$$

Dividing equation (i) by (ii), we have

$$\tan \theta = \frac{v^2}{rg} \quad \text{...(iii)}$$

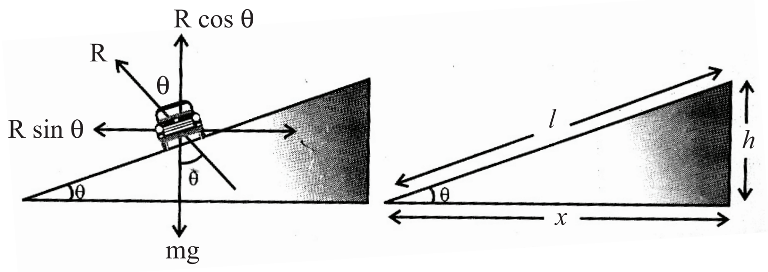
Note :

- For the same reasons, an ice skater or an aeroplane has to bend inwards, while taking a turn.

2.26 Banking of a Road

For getting a centripetal force cyclist bend towards the centre of circular path but it is not possible in case of four wheelers.

Therefore, outer bed of the road is raised so that a vehicle moving on it gets automatically inclined towards the centre.



(A) (B)

$$\tan\theta = \frac{v^2}{rg} \qquad \text{...(iii)}$$

If I = width of the road, h = height of the outer edge from the ground level then from the

$$\tan\theta = \frac{\omega^2 r}{g} = \frac{v\omega}{rg} \qquad [\text{As } v = r\omega] \text{ ...(iv)}$$

If l = width of the road, h = height of the outer edge from the ground level then from the figure (B)

$$\tan\theta = \frac{h}{x} = \frac{h}{l}, \qquad [\text{since } \theta \text{ is very small}] \text{ .... (v)}$$

- Maximum safe speed on a banked frictional road

$$v = \sqrt{\frac{rg(\mu + \tan\theta)}{1 - \mu\tan\theta}}.$$

Kinematics (1 Mark)

1. Under what condition is the average velocity equal the instantaneous velocity ?
2. Draw Position time graph of two objects, A & B moving along a straight line, when their relative velocity is zero.

3. Suggest a situation in which an object is accelerated and have constant speed.

4. Two balls of different masses are thrown vertically upward with same initial velocity. Maximum heights attained by them are h_1 and h_2 respectively what is h_1/h_2 ?

5. A car moving with velocity of 50 kmh^{-1} on a straight road is ahead of a jeep moving with velocity 75 kmh^{-1} . How would the relative velocity be altered if jeep is ahead of car ?

6. Which of the two-linear velocity or the linear acceleration gives the direction of motion of a body ?

7. Will the displacement of a particle change on changing the position of origin of the coordinate system ?

8. If the instantaneous velocity of a particle is zero, will its instantaneous acceleration be necessarily zero ?

9. A projectile is fired with Kinetic energy 1 KJ. If the range is maximum, what is its Kinetic energy, at the highest point ?

10. Write an example of zero vector.

11. State the essential condition for the addition of vectors.

12. When is the magnitude of $(\vec{A} + \vec{B})$ equal to the magnitude of $(\vec{A} - \vec{B})$?

13. What is the maximum number of component into which a vector can be resolved ?

14. A body projected horizontally moves with the same horizontal velocity although it moves under gravity. Why ?

15. What is the angle between velocity and acceleration at the highest point of a projectile motion ?

16. When does (i) height attained by a projectile maximum ? (ii) horizontal range is maximum ?

17. What is the angle between velocity vector and acceleration vector in uniform circular motion ?

18. A particle is in clockwise uniform circular motion the direction of its acceleration is radially inward. If sense of rotation or particle is anticlockwise then what is the direction of its acceleration ?

19. A train is moving on a straight track with acceleration a. A passenger drops a stone. What is the acceleration of stone with respect to passenger ?

20. What is the average value of acceleration vector in uniform circular motion over one cycle ?

21. Does a vector quantity depends upon frame of reference chosen ?

22. What is the angular velocity of the hour hand of a clock ?

23. What is the source of centripetal acceleration for earth to go round the sun ?

24. What is the unit vector perpendicular to the plane of vectors $\vec{A}$ and $\vec{B}$? If $\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 2\hat{i} + \hat{k}$, find a unit vector perpendicular to plane of $\vec{A}$ and $\vec{B}$.

25. What is the angle between $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$?

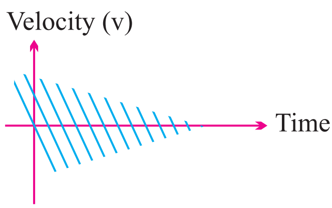
2 Marks

26. What are positive and negative acceleration in straight line motion ?

27. Can a body have zero velocity and still be accelerating ? If yes gives any situation.

28. The displacement of a body is proportional to t^3, where t is time elapsed. What is the nature of acceleration -time graph of the body ?

29. Suggest a suitable physical situation for the following graph.



30. An object is in uniform motion along a straight line, what will be position time graph for the motion of object, if

(i) x_0 = positive, v = negative $|\vec{v}|$ is constant.

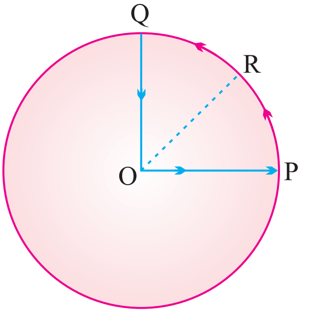
(ii) both x_0 and v are negative $|v|$ is constant.

(iii) x_0 = negative, v = positive $|\vec{v}|$ is constant.

(iv) both x_0 and v are positive $|\vec{v}|$ is constant.

where x_0 is position at $t = 0$.

31. A cyclist starts from centre O of a circular park of radius 1 km and moves along the path OPRQO as shown. If he maintains constant speed of 10 ms^{-1}. What is his acceleration at point R in magnitude & direction ?



32. What will be the effect on horizontal range of a projectile when its initial velocity is doubled keeping angle of projection same ?

33. The greatest height to which a man can throw a stone is h. What will be the greatest distance upto which he can throw the stone ?

34. A person sitting in a train moving at constant velocity throws a ball vertically upwards. How will the ball appear to move to an observer.

(i) Sitting inside the train

(ii) Standing outside the train

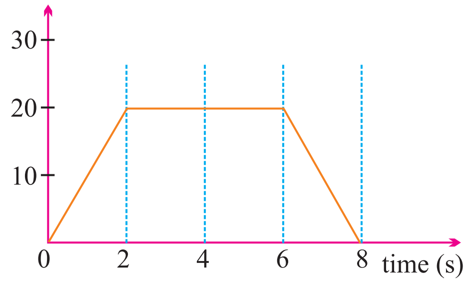
35. A gunman always keep his gun slightly tilted above the line of sight while shooting. Why ?

36. Is the acceleration of a particle in circular motion not always towards the centre. Explain.

3 Marks

37. Draw (a) acceleration - time (b) velocity - time (c) Position - time graphs representing motion of an object under free fall. Neglect air resistance.

38. The velocity time graph for a particle is shown in figure. Draw acceleration time graph from it.



39. For an object projected upward with a velocity v_0 , which comes back to the same point after some time, draw

(i) Acceleration-time graph

(ii) Position-time graph

(iii) Velocity time graph

40. The acceleration of a particle in ms^{-2} is given by $a = 3t^2 + 2t + 2$, where time t is in second.

If the particle starts with a velocity $v = 2\ ms^{-1}$ at $t = 0$, then find the velocity at the end of 2 s.

41. At what angle do the two forces (P + Q) and (P – Q) act so that the resultant is $\sqrt{3P^2 + Q^2}$.

42. Establish the following vector inequalities :

(i) $|\bar{a} + \bar{b}| \leq |\bar{a}| + |\bar{b}|$

(ii) $|\bar{a} + \bar{b}| \geq |\bar{a}| - |\bar{b}|$

43. A body is projected at an angle θ with the horizontal. Derive an expression for its horizontal range. Show that there are two angles θ_1 and θ_2 projections for the same horizontal range, such that $\theta_1 + \theta_2 = 90°$.

44. Prove that the maximum horizontal range is four times the maximum height attained by the projectile, when fired at an inclination so as to have maximum range.

45. Show that there are two values of time for which a projectile is at the same height. Also show that the sum of these two times is equal to the time of flight.

46. A car moving along a straight highway with speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop ?

5 Marks

47. Derive the following equations of motion for an object moving with constant acceleration along a straight line using graphical method and calculas method

(i) velocity time relation ($v = u + at$)

(ii) Position time relation ($s = ut + \frac{1}{2}at^2$)

(iii) velocity-displacement relation ($v^2 = u^2 + 2as$)

where symbols have usual meanings.

48. A projectile is fired horizontally with a velocity u. Show that its trajectory is a parabola. Also obtain expression for

(i) time of flight

(ii) horizontal range

(iii) velocity at any instant.

49. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a particle moving with constant speed v along a circular path of radius r.

Numericals

50. The V-t graphs of two objects make angle 30° and 60° with the time axis. Find the ratio of their accelerations.

51. When the angle between two vectors of equal magnitudes is $2\pi/3$, prove that the magnitude of the resultant is equal to either.

52. If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$, find a vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$.

53. (a) If $\hat{i}$ and $\hat{j}$ are unit vectors along x & y axis respectively then what is magnitude and direction of $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$?

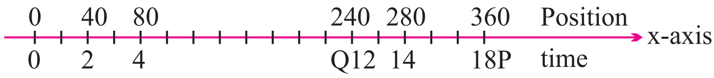
(b) Find the components of vector $\vec{a} = 2\hat{i} + 3\hat{j}$ along the directors of vectors $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$.

54. What is the vector sum of n coplanar forces, each of magnitude F, if each force makes an angle of $\frac{2\pi}{n}$ with the preceding force ?

55. A car is moving along x-axis. As shown in figure it moves from O to P in 18 seconds and return from P to Q in 6 second. What are the average velocity and average speed of the car in going from

(i) O to P

(ii) from O to P and back to Q



56. On a 60 km straight road, a bus travels the first 30 km with a uniform speed of 30 kmh^{-1} . How fast must the bus travel the next 30 km so as to have average speed of 40 kmh^{-1} for the entire trip ?

57. The displacement x of a particle varies with time as $x = 4t^2 - 15t + 25$. Find the position, velocity and acceleration of the particle at $t = 0$.

58. A driver take 0.20 second to apply the breaks (reaction time). If he is driving car at a speed of 54 kmh^{-1} and the breaks cause a decleration of 6.0 ms^{-2}. Find the distance travelled by car after he sees the need to put the breaks.

59. From the top of a tower 100 m in height a ball is dropped and at the same time another ball is projected vertically upwards from the ground with a velocity of 25 m/s. Find when and where the two balls will meet ? (g = 9.8 m/s)

60. A ball thrown vertically upwards with a speed of 19.6 ms^{-1} from the top of a tower returns to the earth in 6s. Find the height of the tower. (g = 9.8 m/s^2)

61. Two town A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20 kmh^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction.

What is the period T of the bus service and with what speed do the busses ply of the road ?

62. A motorboat is racing towards north at 25 kmh^{-1} and the water current in that region is 10 kmh^{-1} in the direction of 60° east of south. Find the resultant velocity of the boat.

63. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft position 10 second apart is 30°, what is the speed of the aircraft ?

64. A boat is moving with a velocity $(3\hat{i} + 4\hat{j})$ with respect to ground. The water in river is flowing with a velocity $(-3\hat{i}-4\hat{j})$ with respect to ground. What is the relative velocity of boat with respect to river ?

65. A hiker stands on the edge of a clift 490 m above the ground and throws a stone horizontally with an initial speed of 15 ms^{-1}. Neglecting air resistance, find the time taken by the stone to reach the ground and the speed with which it hits the ground. ($g = 9.8$ ms^{-2})

66. A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting the angle of projection, can one hope to hit the target 5 km away ? Assume that the muzzle speed to be fixed and neglect air resistance.

67. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 seconds, what is the magnitude and direction of acceleration of the stone ?

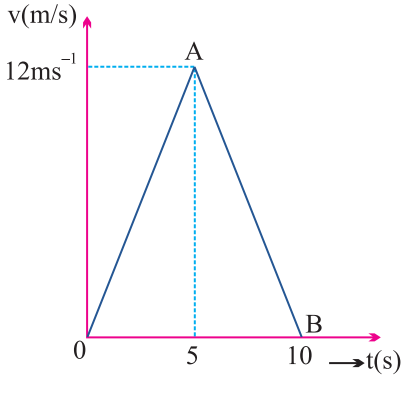
68. A cyclist is riding with a speed of 27 kmh^{-1} . As he approaches a circular turn on the road of radius 30 m, he applies brakes and reduces his speed at the constant rate 0.5 ms^{-2}. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn ?

69. If the magnitude of two vectors are 3 and 4 and their scalar product is 6, find angle between them and also find $|\vec{A}\times\vec{B}|$.

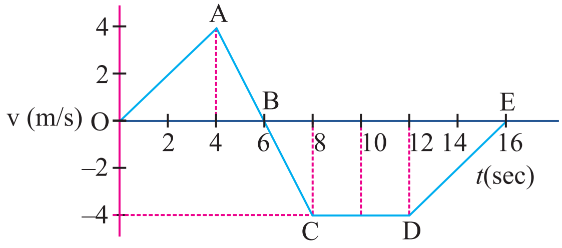
70. Find the value of λ so that the vector $\vec{A}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{B}=4\hat{i}-2\hat{j}+2\hat{k}$ are perpendicular to each other.

71. The speed-time graph of a particle along a fixed direction is as shown in Fig. obtain the distance travelled by a particle between (a) t = 0 to 10s, (b) t = 2 to 6 sec.

What is the average speed of the particle over the intervals in (a) and (b) ?



72. The velocity time graph of a particle is given by



(i) Calculate distance and displacement of particle from given *v-t* graph.
(ii) Specify the time for which particle undergene acceleration, retardation and moves with constant velocity.
(iii) Calculate acceleration, retardation from given *v-t* graph.
(iv) Draw accleration-time graph of given *v-t* graph.

73. If $\vec{a}, \vec{b}$ and $\vec{c}$ are represented by three sides of triangle taken in same order.

P. T. $\vec{a}+\vec{b}+\vec{c} = \vec{0}$. **[Hint :** use Δ law of vector Addition]

74. Prove that for any vector $\vec{a}$,

$$\vec{a} = (\vec{a}.\hat{i})\hat{i} + (\vec{a}.\hat{j})\hat{j} + (\vec{a}.\hat{k})\hat{k},$$

75. If R is the horizontal range for θ inclination and h is the maximum height attained by the projectile, show that the maximum range is given by

$\frac{R^2}{8h} + 2h.$ **[Hint :** Put value in relation given & solve]

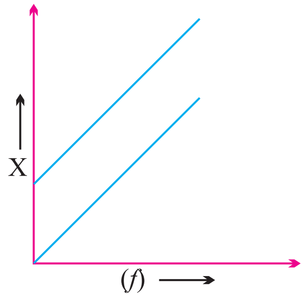
76. The Resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If the direction of one of the vector is reversed, then resultant becomes $\vec{S}$.

Prove that $R^2 + S^2 = 2(P^2 + Q^2)$.

SOLUTIONS

1. When the body is moving with uniform velocity. **[1 Mark Solutions]**

2.



3. Uniform Circular Motion

4. Same height, $\therefore h_1/h_2 = 1$

5. No change

6. Linear velocity

7. will not change.

8. No. (highest point of vertical upward motion under gravity)

9. here $\frac{1}{2}mv^2 = 1$ KJ $= 1000$ J, $\theta = 45°$

At the highest Point, K. E. $= \frac{1}{2}m(v\cos\theta)^2 = \frac{1}{2}\frac{mv^2}{2} = \frac{1000}{2} = 500$ J.

10. The velocity vectors of a stationary object is a zero vectors.

11. They must represent the physical quantities of same nature.

12. When $\vec{A}$ is perpendicular to $\vec{B}$.

13. Infinite.

14. Because horizontal component of gravity is zero along horizontal direction.

15. 90°.

16. height is maximum at $\theta = 90$

Range is maximum at $\theta = 45$.

17. 90°

18. Radial inward.

19. $\sqrt{a^2 + g^2}$ where g = acceleration due to gravity.

20. Null vector

21. No.

22. $\omega = \frac{2\pi}{12} = \frac{\pi}{6}$ rad h^{-1}.

23. Gravitation force of sun.

24. $\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$, $\hat{n} = \frac{2\hat{i} - 3\hat{j} - 4\hat{k}}{\sqrt{29}}$.

25. 90°

Answers

[2 Marks]

26. If speed of an object increases with time, its acceleration is positive. (Acceleration is in the direction of motion) and if speed of an object decreases with time its acceleration is negative (Acceleration is opposite to the direction of motion).

27. Yes, at the highest point of vertical upward motion under gravity.

28. as $s \propto t^3 \Rightarrow s = kt^3$

$$\text{velocity, } V = \frac{ds}{dt} = 3kt^2$$

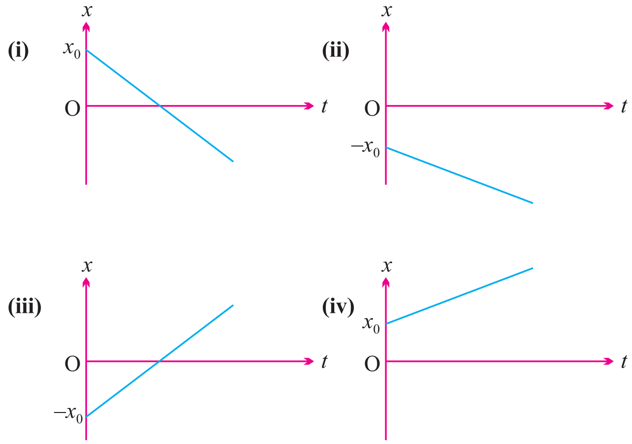
$$\text{acceleration, } a = \frac{dv}{dt} = 6kt$$

i.e., $a \propto t$.

$\Rightarrow$ motion is uniform, accelerated motion, $a - t$ graph is straight-line.

29. A ball thrown up with some initial velocity rebounding from the floor with reduced speed after each hit.

30.



31. Centripetal acceleration, $a_c = \frac{v^2}{r} = \frac{10^2}{1000} = 0.1 \text{ m/s}^2$ along RO.

32. $R = \frac{u^2 \sin 2\theta}{g} \Rightarrow R \propto u^2$

Range becomes four times.

33. Maximum height ; $H = \frac{u^2 \sin^2 \theta}{2g}$

$$\Rightarrow \quad H_{max} = \frac{u^2}{2g} = h (\text{at } \theta = 90°)$$

Max. Range $R_{max} = \frac{u^2}{g} = 2h$.

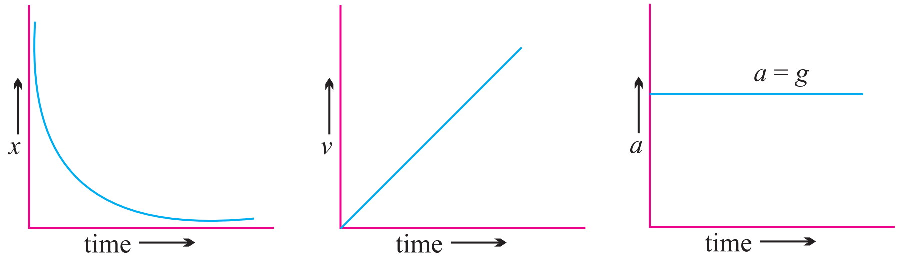
34. (i) Vertical straight line motion

(ii) Parabolic path.

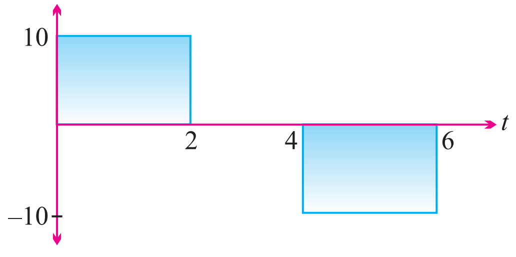
35. Because bullet follow Parabolic trajectory under constant downward acceleration.

36. No acceleration is towards the centre only in case of uniform circular motion.

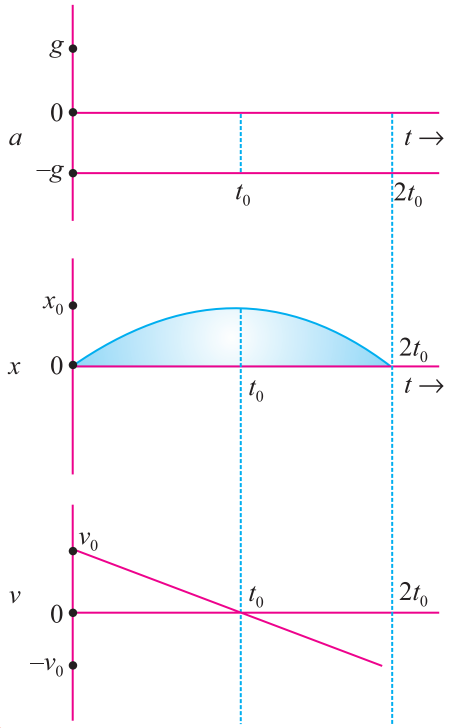
37. The object falls with uniform acceleration equal to 'g'



38.



39.



$2t_0 \rightarrow$ total time of the Journey

$x_0 \rightarrow$ highest position

40. $a = \dfrac{dv}{dt} = 3t^2 + 2t + 2)\ dt$

$dv = (3t^2 + 2t + 2)dt$

$\int dv = \int (3t^2 + 2t + 2)dt$

$v = t^3 + t^2 + 2t + c$

θ

$c = 2$ m/s, $v = 18$ m/s at $t = 2s$.

41. Use $R = \sqrt{A^2 + B^2 + 2AB\cos Q}$

$R = \sqrt{3P^2 + Q^2}, A = P + Q,$

$B = P - Q$

solve, $\theta = 60°$

46. Initial velocity of car,

$$u = 126 \text{ kmh}^{-1} = 126 \times \frac{5}{18} \text{ ms}^{-1} = 35 \text{ ms}^{-1} \quad \text{...(i)}$$

Since, the car finally comes to rest, $v = 0$

Distance covered, $s = 200$ m, $a = ?$, $t = ?$

$v^2 = u^2 - 2as$

or $a = \dfrac{v^2 - u^2}{2s}$ ...(ii)

Substituting the values from eq. (i) in eq. (ii), we get

$$a = \frac{0 - (35)^2}{2 \times 200} = -\frac{35 \times 35}{400}$$

$$= -\frac{49}{16} \text{ ms}^{-2}$$

$$= -3.06 \text{ ms}^{-2}$$

Negative sign shows that acceleration in negative which is called retardation, *i.e.*, car is uniformly retarded at $-a = 3.06$ ms^{-2}.

To find t, let us use the relation

$v = u + at$

$t = \dfrac{v - u}{a}$

Use $a = -3.06$ ms^{-2}, $v = 0$, $u = 35$ ms^{-1}.

$\therefore \qquad t = \frac{v-u}{a} = \frac{0-35}{-3.06} = 11.44$ s

$\therefore \qquad t = 11.44$ sec.

NUMERICALS

Answer

50. $\frac{a_1}{a_2} = \frac{\tan 30}{\tan 60} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3} = 1:3$

51. $R = (P^2 + Q^2 + 2PQ\cos\theta)^{1/2}$

$= \left(P^2 + P^2 + 2P.P\cos\frac{2\pi}{3}\right)^{1/2}$

$= \left[2P^2 + 2P^2\left(\frac{-1}{2}\right)\right]^{1/2} = P$

52. $|\vec{A}| = \sqrt{3^2 + 4^2} = 5$

also $\qquad |\vec{B}| = \sqrt{7^2 + 24^2} = 25$

desired vector $= |\vec{B}|\hat{A} = 25 \times \frac{3\hat{i} + 4\hat{j}}{5}$

$= 5(3\hat{i} + 4\hat{j}) = 15\hat{i} + 20\hat{j}$.

54. Resultant force is zero.

55. (i) O to P, Average velocity = 20 ms^{-1}

(ii) O to P and back to Q

Average velocity = 10 ms^{-1}

Average speed = 20 ms^{-1} $\qquad V_2$

56. $V_{avg} = \frac{S_1 + S_2}{t_1 + t_2} = \frac{S+S}{S\left(\frac{1}{V_1} + \frac{1}{V_2}\right)} = \frac{2V_1V_2}{V_1 + V_2}$

or $\qquad 40 = \frac{2 \times 30 \times v_2}{V_1 + V_2} \Rightarrow V_2 = 60$ kmh^{-1}

57. position, $x = 25$ m

velocity $= \frac{dx}{dt} = 8t - 15$,

$t = 0$, $v = 0 - 15 = -15$ m/s

acceleration, $a = \frac{dv}{dt} = 8\ ms^{-2}$.

58. (distance covered during 0.20 s) + (distance covered until rest)

$=(15 \times 0.25) + [18.75] = 21.75$ m

59. For the ball chapped from the top

$x = 4.9t^2$ ...(i)

For the ball thrown upwards

$100 - x = 25t - 4.9t^2$ ...(ii)

From eq. (i) & (ii), 36

$t = 4$s; x = 78.4 m

60. using $s = ut + \frac{1}{2}at^2$

$-h = 19.6 \times 6 + \frac{1}{2} \times (-9.8) \times 62$

$h = 58.8$ m.

61. $V = 40\ kmh^{-1}$ and T = 9 min.

62. $V = 21.8\ kmh^{-1}$

angle with north, $\theta = 23.4$

63. Speed = $182.2\ ms^{-1}$

64. $\vec{V}_{BW} = \vec{V}_B - \vec{V}_W$

$\vec{V}_{BW} = 6i + 8j.$

65. time = 10 seconds

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{15^2 \quad 98^2} = 99.1\ m/s^{-1}$.

66. Maximum Range = 3.46 km

So it is not possible.

67. $\omega = \frac{88}{25} rad\, s^{-1}$, $\omega = \frac{2\pi}{T} = \frac{2\pi N}{t}$

$a = \quad c + \quad T \quad 991.2\ cms^{-2}$

68. $a_c = \frac{v^2}{r} = 0.7\ ms^{-2}$

$a_T = 0.5\ ms^{-2}$

$$a = \sqrt{2^2{}_C + a^2{}_T} = 0.86 \text{ ms}^{-2}$$

If θ is the angle between the net acceleration and the velocity of the cyclist, then

$$\theta = \tan^{-1}\left(\frac{a_c}{a_T}\right) = \tan^{-1}(1.4) = 54° \, 28'$$

69. $\vec{A}.\vec{B} = AB \cos\theta$, $\quad |\vec{A} \times \vec{B}| = AB \sin\theta$

or $\quad 6 = (3 \times 4) \cos\theta \quad = 3 \times 4 \times \sin 60°$

or $\quad \theta = 60° \quad = 3 \times 4 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$

70. $\because \quad \vec{A} \perp \vec{B} \Rightarrow \vec{A}.\vec{B} = 0 \quad \because \; 8 - 2\lambda + 2 = 0$

$\Rightarrow \quad \boxed{\lambda = 5.}$

71. Refer NCERT.

72. (i) distance = area of ΔOAB + area of trapezium BCDE

$= 12 + 28 = 40$ m

(ii) displacement = area of ΔOAB – area of trapezium BCDE

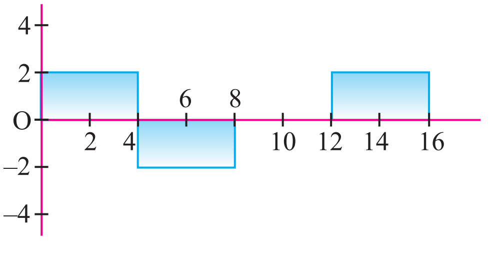
$= 12 - 28 = -16$ m

(iii) time acc. $(0 \le t \le 4)$ and $(12 \le t \le 16)$

retardation $(4 \le t \le 8)$

constant velocity $(8 \le t \le 12)$

(iv)



KINEMATICS (M.C.Q.)

1. If the angle between the vectors $\vec{A}$ and $\vec{B}$ is θ, the value of the product $(\vec{B}\times\vec{A})\cdot\vec{A}$ is equal to

   (a) $BA^2\cos\theta$ (b) $BA^2\sin\theta$

   (c) $BA^2\sin\theta\cos\theta$ (d) Zero

2. For an object thrown at 45° to the horizontal, the maximum height (H) and horizontal range (R) are related as

   (a) R = 16 H (b) R = 8 H

   (c) R = 4 H (d) R = 2 H

3. The circular motion of a particle wth constant speed is

   (a) Simple harmonic but not periodic

   (b) Periodic an simple harmonic

   (c) Neither periodic nor simple harmonic

   (d) Periodic but not simple harmonic

4. At the upper most of a projectile, its velocity and acceleration at an angle of

   (a) 0° (b) 45°

   (c) 90° (d) 180°

5. If for any two vectors $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$ in a plane, then what is angle between $\vec{A}$ and $\vec{B}$

   (a) $\theta = 0$ (b) $\theta = 45°$

   (c) $\theta = 90°$ (d) $\theta = 180°$

6. The x and y coordinates of a particle at any time t is given by $x = 7t + 4t^2$ and $y = 5t$, where x and y are in metre and t in seconds. The acceleration of particle at t = 5s is

   (a) Zero (b) 8 m/s^2

   (c) 20 m/s^2 (d) 40 m/s^2

7. If K is the kinetic energy of a projectile fired at an angle 45°, then what is the kinetic energy at the highest point.

(a) $\frac{K}{4}$ (b) $\frac{K}{2}$

(c) K (d) 2K

8. A particle is moving eastwards with a velocity of 5 m/s. In 10s, the velocity changes to 5 m/s north words. The average acceleration in this time is

(a) Zero (b) $\frac{1}{\sqrt{2}}$ m/s^2 towards north-west

(c) $\frac{1}{2}$ m/s^2 towards north (d) $\frac{1}{\sqrt{2}}$ m/s^2 towards north-east

9. A body dropped from top of tower falls through 60 m during the last 2 seconds of its fall. The height of tower is (g = 10 m/s^2)

(a) 95 m (b) 80 m

(c) 90 m (d) 60 m

10. The angular velocity of seconds hand of a watch is

(a) 60 π rad/s (b) $\frac{\pi}{60}$ π rad/s

(c) 40 π rad/s (d) $\frac{\pi}{30}$ π rad/s

11. The angle between the vectors $(\hat{i}+\hat{j})$ and $(\hat{j}+\hat{k})$ is

(a) 30° (b) 45°

(c) 60° (d) 90°

12. If the scalar and vector products of two vectors $\vec{A}$ and $\vec{B}$ are equal in magnitude, then the angle between the two vectors is

(a) 45° (b) 90°

(c) 180° (d) 120°

13. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by

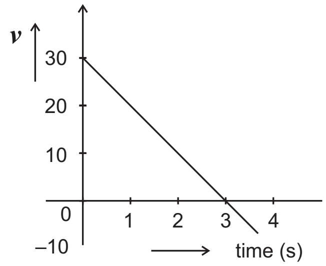
$$\frac{dv}{dt} = -2.5\sqrt{v}$$

Where V is the instantaneous speed. The time taken by the object, to come to rest would be

(a) 1 s (b) 2 s

(c) 4 s (d) 8 (s)

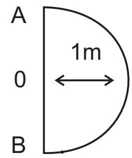
14. The velocity-time graph for the vertical component of the velocity of a body thrown upwards from the ground and landing on the roof of a building is given in the figure. The height of the building is



(a) 50 m (b) 40 m

(c) 20 m (d) 30 m

15. In 1 s, a particle goes from point A to point B moving in a semicircle as shown in figure. The magnitude of the average velocity is



(a) Zero (b) 1 m/s

(c) 2 m/s (d) 3.14 m/s

16. A boat which has speed of 5 km/h in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of rive water in km/hr is

(a) 1 (b) 3

(c) 4 (d) $\sqrt{41}$

17. A body projected at an angle with the horizontal has a range 300 m. I the time of flight is 6s, then the horizontal component of velocity is

(a) 30 m/s (b) 50 m/s

(c) 40 m/s (d) 45 m/s

18. Projection of $\vec{A}$ on $\vec{B}$ is

(a) $\vec{A} \cdot \vec{B}$ (b) $\vec{A} \cdot \hat{B}$

(c) $\vec{A} \times \hat{B}$ (d) $\hat{A} \times \vec{B}$

19. A boy runs along a straight path for the first half distance with a velocity V_1 and second half with velocity V_2. The mean velocity V is given by

(a) $\frac{2}{V} = \frac{1}{v_1} + \frac{1}{v_2}$ (b) $V = \frac{V_1 + v_2}{2}$

(c) $V = \sqrt{v_1 . v_2}$ (d) $\vec{v}_1 + \vec{V}_2$

20. A projectile rises to a height of 10 m and then falls at a distance of 30 m away from the projection. Its vertical displacement is

(a) 0 m (b) 5 m

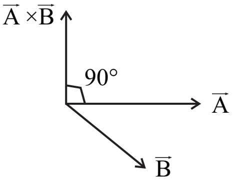
(c) 6 m (d) 7 m

Answer Key :

1. (d)	2. (c)	3. (d)	4. (c)	5. (c)	6. (b)
7. (b)	8. (b)	9. (b)	10. (d)	11. (c)	12. (a)
13. (b)	14. (b)	15. (c)	16. (b)	17. (b)	18. (b)
19. (b)	20. (a)				

HINTS AND EXPLANATIONS :

1. Angle between $\vec{B}\times\vec{A}$ and $\vec{A}$ is 90°



$(\vec{B}\times\vec{A}).\ \vec{A} = |\vec{B}\times\vec{A}|\ |\vec{A}| \cos 90°$

$= 0$

2. $H = \dfrac{U^2 \sin^2 45°}{2g} = \dfrac{U^2}{4g}$,

$R = \dfrac{U^2 \sin 90°}{g} = \dfrac{U^2}{g}$

$\therefore\ R = 4H$

4. At highest pt, only horizontal component of velocity exists, while acceleration is directed downward, So $\theta = 90°$.

5.

$|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

S. B. S.

$|\vec{A}+\vec{B}|^2 = |\vec{A}-\vec{B}|^2$

$(\vec{A}+\vec{B})\bullet(\vec{A}+\vec{B}) = (\vec{A}-\vec{B})\bullet(\vec{A}-\vec{B})$

$\therefore \theta = 90°$

6.

$$x = 7t + 4t^2$$

$$V_x = \frac{dx}{dt} = 7 + 8t$$

$$a_x = \frac{dv_x}{dt} = 8$$

$$y = 5t$$

$$V_y = \frac{dy}{dt} = 5$$

$$a_y = \frac{dv_y}{dt} = 0\ m/s^2$$

$$a = \sqrt{a_x{}^2 + a_y{}^2} = \sqrt{8^2 + 0}$$

$$= 8\ m/s^2$$

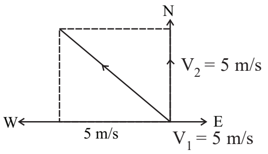
7. $$k = \frac{1}{2} mv^2$$

At highest pt., v = v cos 45°

$$K.E = \frac{1}{2} m\left(\frac{v}{\sqrt{2}}\right)^2 \qquad = \frac{v}{\sqrt{2}}$$

$$= \frac{1}{2}\ \frac{mv^2}{2} = \frac{K}{2}$$

8.



$$acc. = \frac{\Delta v}{t}$$

$$= \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}\ m/s^2$$

$\vec{V}_1 = 5\ m/s$ (East)

$\vec{V}_2 = 5\ m/s$ (North)

Change in velocity $= \vec{V}_t - \vec{V}_i$

$$= 5\hat{j} - 5\hat{i}\ m/s$$

Δv = Magnitude of change in velocity

$$= \sqrt{(5)^2 + (5)^2} = 5\sqrt{2}\ m/s$$

9. U = O, velocity attained by ball at t = 2 second

v = u + a t = 0 + g (t – 2)

distance traveled by ball in last 2s is given by

$$h_1 = vt + \frac{1}{2} a\, t^2$$

$$60 = g(t-2) \times 2 + \frac{1}{2} g(4) = 20(t-2) + 20$$

at 4S, Height of tower

$$= \frac{1}{2} g\, t^2 = \frac{1}{2} \times 10 \times 4^2 = 80\ m$$

10. $$w = \frac{2\pi}{T} = \frac{2\pi}{60} = \frac{\pi}{30}\ \text{rod/s}$$

11. $$\cos\theta = \frac{\vec{A} \bullet \vec{B}}{|\vec{A}||\vec{B}|}$$

$$= \frac{(\hat{i} + \hat{j}) \bullet (\hat{j} + \hat{k})}{\sqrt{1+1}\ \sqrt{1+1}} = \frac{0+1+0}{2}$$

$$\cos\theta = \frac{1}{2} = \cos 60°$$

$$\theta = 60°$$

12. $\left|\vec{A}\bullet\vec{B}\right| = \left|\vec{A}\times\vec{B}\right|$

$AB\cos\theta = AB\sin\theta$

$1 = \tan\theta$

$\tan\theta = 1$

$\tan\theta = \tan 45°$

$\boxed{\theta = 45°}$

13. $\dfrac{dv}{dt} = -2.5\sqrt{v}$

$\int_{6.25}^{0} v^{-1/2}\, dv = \int_{0}^{t} 2.5\, dt$

$\left[\dfrac{v^{½}}{½}\right]_{6.25}^{0} = 2.5\ [t]_0^t$

$2\left[0 - \sqrt{6.25}\right) = -2.5\,(t-0)$

$2\left[-2.5\right) = -2.5\,t$

$t = 2s$

14. $u = 30$ m/s

$v = 0$ m/s

$t = 3s \qquad S = h = ?$

$v = u + a\,t$

$0 = 30 + a\,(3)$

$a = \dfrac{-30}{3} = -10\ m/s^2$

$V^2 = u^2 + 2a\,S$

$O = (30)^2 + 2(-10)\,h$

$-(30)^2 = -20\,h$

$h = \dfrac{-30\times 3\cancel{0}}{-2\cancel{0}} = 45m$

From Max. height to roof of building

$h = \dfrac{u^2}{2g} = \dfrac{10^2}{2\times 10} = 5m$

height of building $= H - h = 45 - 5$

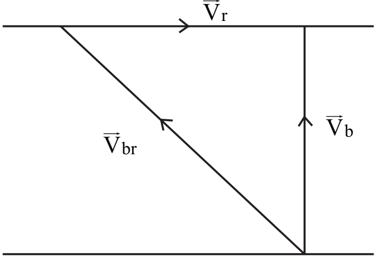
$= 40$ m

15. Magnitude of average

velocity $= \dfrac{\text{Magnitude of displacement}}{\text{Time taken}}$

$= \dfrac{2m}{1s} = 2ms^{-1}$

16.



$$\vec{V}_{br} = \vec{V}_b - \vec{V}_r$$

$$V_{br}^2 = V_b^2 + V_r^2$$

$$V_b^2 = 25 - V_r^2 \text{ ———— (1)}$$

$$t = \frac{\text{distance}}{V_b} \Rightarrow$$

$$\frac{1}{4} \text{hr} = \frac{1}{\sqrt{25 - V_r^2}}$$

$$V_r = 3 \text{ km/h}$$

17. R = (U cos θ)T

$$U\cos\theta = \frac{R}{T} = \frac{300}{6} = 50 \text{ m/s}$$

18. Projection of $\vec{A}$ on $\vec{B}$

$$= \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|} = A \cdot \frac{\vec{B}}{|\vec{B}|} = \vec{A} \cdot \hat{B}$$

19. Ar. Velocity

$$V = \frac{S_1 + S_2}{t} = \frac{\frac{V_1 t}{2} + \frac{V_2 t}{2}}{t}$$

$$V_1 = \frac{S_1}{t/2}, \quad V_2 = \frac{S_2}{t/2}$$

$$= \frac{V_1 + V_2}{2}$$

20. Since the projectile returns to the plane of projection therefore the net displacement is zero.

****